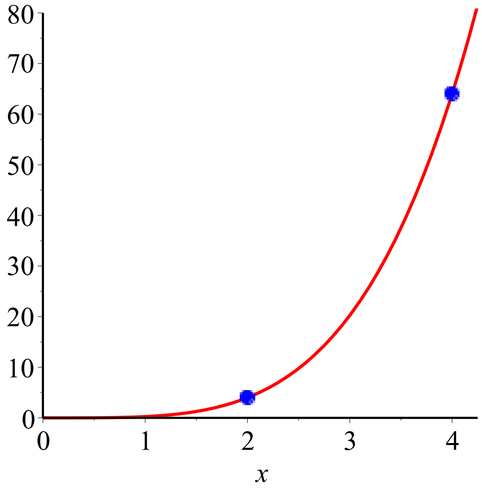
Power function

$f(x) = b \cdot x^a$

$b > 0$
For $a < 0$, $f(x)$ is decreasing,
For $0 < a$, $f(x)$ is increasing.

$Dm(f) = \mathbb{R}_+$ $\quad$ $Vm(f) = \mathbb{R}_+$

$f(x) = 0,25 \cdot x^4$



If x is increasing by the factor h, then $f(x)$ will increase by the factor h^a

(*When* x is increasing by a specific %, then $f(x)$ is also increasing by a specific %)

In the example above : $f(x) = 0,25 \cdot x^4$, $f(x)$ is increasing by the factor $h^a = 2^4 = 16$

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$$f(h \cdot x) = b \cdot (h \cdot x)^a = b \cdot h^a \cdot x^a = h^a \cdot \left(b \cdot x^a\right) = h^a \cdot f(x)$$

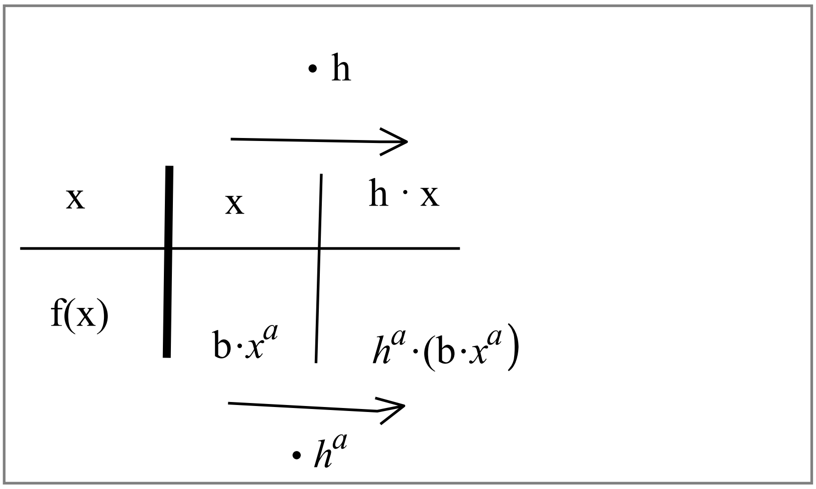
□

$$f(h \cdot x) = h^a \cdot f(x) \qquad (1)$$

Using (1) , we get:

$$\Delta f = f(h \cdot x) - f(x) = h^a \cdot f(x) - f(x) = f(x) \cdot [h^a - 1] \ , \quad \Delta f = f(x) \cdot [h^a - 1] \quad (2)$$

Below graphic illustration of (1), shows power growth:



***example* 1 :** The braking distance of a car $L = b \cdot v^2$ $[\ f(x) = b \cdot x^2\ ]$

How many % will the braking distance increase, when v increase by *10 % ?*

v is increasing by 10 %: $h = 1,10$

L then increases by: $h^a = 1,10^2 = 1,21$, ie. L is increasing by 21 %

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***example* 2 :** $f(x) = b \cdot x^a$, $f(2) = 4$ and $f(4) = 64$ *ie.* $h = 2$

When x increases from 2 to 4 , $f(x)$ is increasing from 4 to 64.

ie. when x is increasing by the factor h $= 2$,

then $f(x)$ will increase by the factor $h^a = 16$ $(4 \cdot 16 = 64)$

It corresponds to: When x is increasing by $(h - 1) \cdot 100\ \% = (2 - 1) \cdot 100\ \% = 100\ \%$,

Then $f(x)$ is increasing by $(h^a - 1) \cdot 100\ \% = (16 - 1) \cdot 100\ \% = 1500\ \%$, as seen in (2)

Calculation of a and b:

$f(x) = b \cdot x^a$, $f(2) = 4$ and $f(4) = 64$

$$a = \frac{\log\left(\frac{y_2}{y_1}\right)}{\log\left(\frac{x_2}{x_1}\right)} = \frac{\ln\left(\frac{y_2}{y_1}\right)}{\ln\left(\frac{x_2}{x_1}\right)} = \frac{\ln\left(\frac{64}{4}\right)}{\ln\left(\frac{4}{2}\right)} = \frac{\ln(16)}{\ln(2)} = \frac{\ln(2^4)}{\ln(2)} = 4 \cdot \frac{\ln(2)}{\ln(2)} = 4$$

$$f(x) = b \cdot x^4 \Leftrightarrow f(2) = b \cdot 2^4 \Leftrightarrow 4 = b \cdot 16 \Leftrightarrow b = \frac{4}{16} = \frac{1}{4} ,$$

ie. $a = 4$ and $b = \frac{1}{4}$, $f(x) = \frac{1}{4} \cdot x^4$