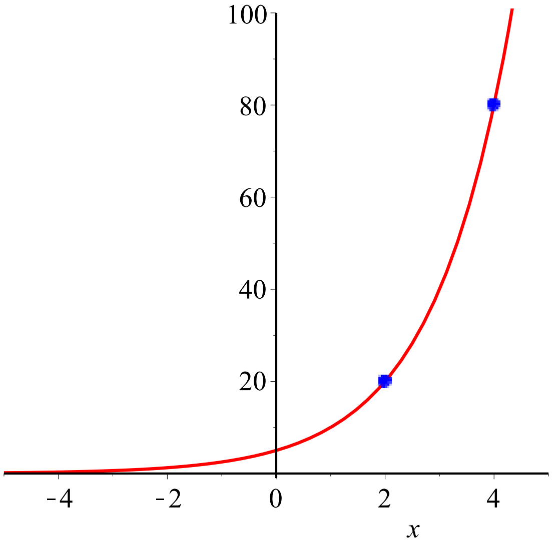
Exponential function:

$f(x) = b \cdot a^x$

$b > 0$
For $0 < a < 1$ $f(x)$ is decreasing
For $1 < a$ $f(x)$ is increasing.

$Dm(f) = \mathbb{R}$ $Vm(f) = \mathbb{R}_+$

$f(x) = 5 \cdot 2^x$



If x is increasing by Δx , then $f(x)$ *is multipliet* by $a^{\Delta x}$

(If x is increasing by a specific size Δx , then $f(x)$ is increasing by a specific %)

a = 1 + r, the number a is called the base.
$r = a - 1$ is called the growth rate.

In the example above: $f(x) = 5 \cdot 2^x$, $f(x)$ is multipliet by $a^{\Delta x} = 2^2 = 4$

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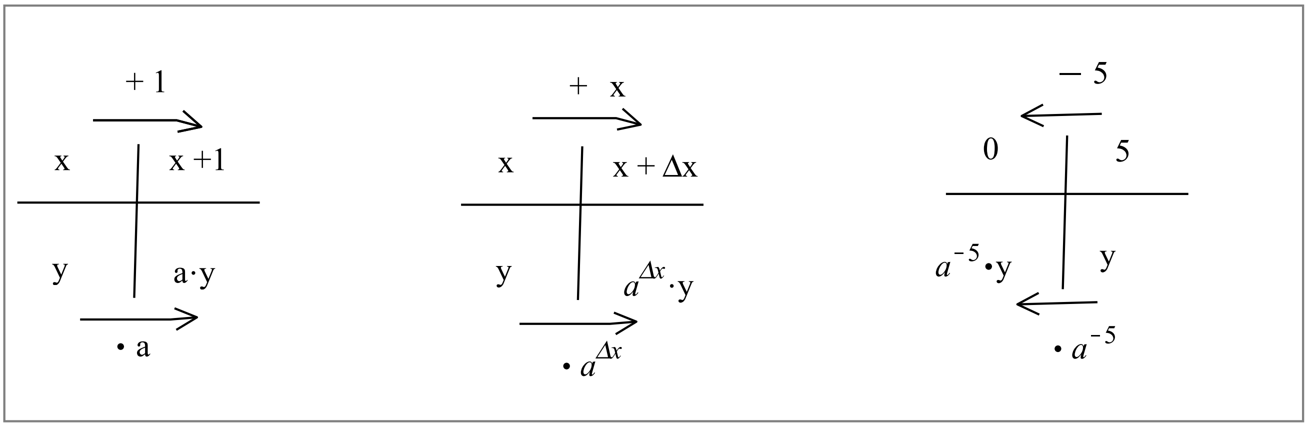
$$f(x + \Delta x) = b \cdot a^{x + \Delta x} = b \cdot a^x \cdot a^{\Delta x} = a^{\Delta x} \cdot (b \cdot a^x) = a^{\Delta x} \cdot f(x)$$

□

$$f(x + \Delta x) = a^{\Delta x} \cdot f(x)$$

From page 1, we have: $f(x+\Delta x) = a^{\Delta x} \cdot f(x)$ (1)

Below graphic illustration shows exponential growth:



From (1) we can deduce the growth Δf the function gets, by a growth Δx:

$$\Delta f = f(x+\Delta x) - f(x) = a^{\Delta x} \cdot f(x) - f(x) = f(x)\left[a^{\Delta x} - 1\right], \quad \Delta f = f(x)\left[a^{\Delta x} - 1\right]$$

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Or to put it another way: (we use the function $f(x) = 5 \cdot 2^x$ as an example.)

When x is growing by $\Delta x = 2$, $f(x)$ is growing by
$(a^{\Delta x} - 1) \cdot 100\ \% = (4 - 1) \cdot 100\ \% = 300\ \%$

[in this projection $f(x) \cdot a^{\Delta x}$, the projection base $a^{\Delta x} = 1 + r \Leftrightarrow r = a^{\Delta x} - 1$.
and r *in* % , is precisely $r \cdot 100\ \% = (a^{\Delta x} - 1) \cdot 100\ \%$]

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Calculation of *a* and *b* :

$f(x) = b \cdot a^x$, $f(2) = 20$ and $f(4) = 80$

$$a = \sqrt[x_2 - x_1]{\frac{y_2}{y_1}} = \left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2 - x_1}} = \left(\frac{80}{20}\right)^{\frac{1}{4-2}} = 4^{\frac{1}{2}} = \sqrt{4} = 2$$

$$f(x) = b \cdot 2^x \Leftrightarrow f(2) = b \cdot 2^2 \Leftrightarrow 20 = b \cdot 4 \Leftrightarrow b = \frac{20}{4} = 5$$

ie. $a = 2$ and $b = 5$; $f(x) = 5 \cdot 2^x$